Gravitational Memory Charges of Supertranslation and Superrotation on Rindler Horizons

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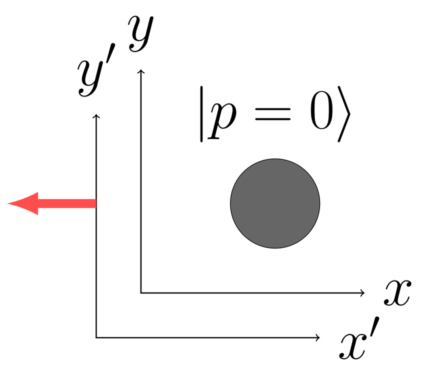
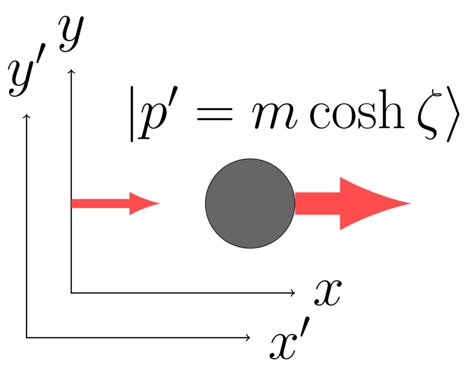
1) Supertranslation (ST) and Superrotations (SR)

Soft hair of black holes comes from would-be gauge degrees of freedom of general covariance (diffeomorphisms)

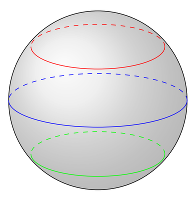
This is in a similar way like when Lorentz transformations generate an infinite number of physical states with different values of momentum.



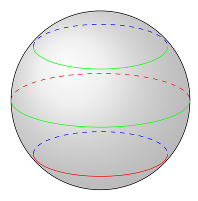
$$t = t'\cosh\zeta + x'\sinh\zeta$$
$$x = x'\cosh\zeta + t'\sinh\zeta$$
$$\langle p=0|\, p' = m\cosh\zeta\rangle = 0$$



Asymptotic symmetries on the horizon generate physical states with different values of holographic charges.



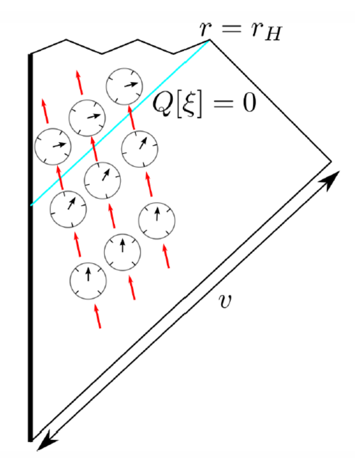
ST: $\tau' = \tau + T(\theta,\phi)$
SR: $\theta' = \Theta(\theta,\phi)$
$\phi' = \Phi(\theta,\phi)$



$|Q=0\rangle$ $\quad\langle Q=0|\,Q'=q\rangle = 0\quad$ $|Q'=q\rangle$

2) Hawking, Perry, Strominger research and ours

HPS results: PRL 116, 231301 (2016)

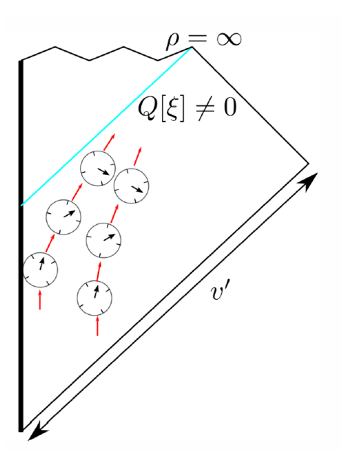


Stationary black holes do not carry classical supertranslation hair on horizon

$$ds^2 = 2dvdr + g_{AB}\, dx^A dx^B + \mathcal{O}(r - r_H)$$

Near the horizon this coordinate system may be implemented by free-falling clocks, which play a role of metric detector (soft hair detector)

Hotta, Sasaki, Sasaki, CQG 18, 1823 (2001)

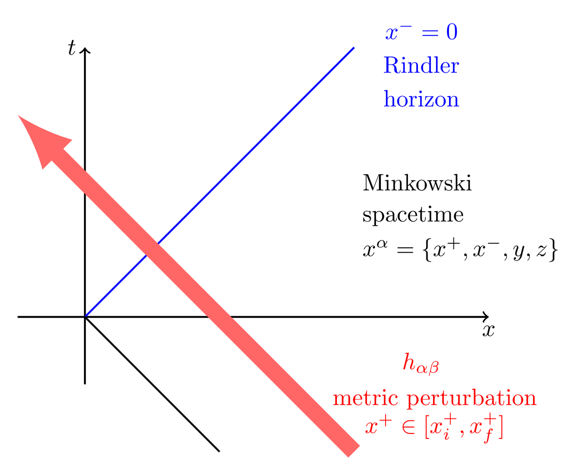


Stationary black holes do carry classical supertranslation and superrotation charges

$$ds^2 = 2\exp\left(-\tfrac{\rho}{\kappa}\right)dx^+d\rho + g_{AB}\, dx^A dx^B + ...$$

This coordinate system may be implemented by accelerating clocks distributed in the spacetime

Our model: PRD 94 083001 (2016)



In Rindler coordinates $\sigma^\mu = \{\tau, \rho, y, z\}$

$$ds^2 = \Delta\left(-d\tau^2 + d\rho^2\right) + dy^2 + dz^2 + \varphi_{\mu\nu}d\sigma^\mu d\sigma^\nu$$

where $\Delta = \exp\left(-\frac{\rho}{\kappa}\right) = \frac{x^+x^-}{4\kappa^2}$

Horizon at $\rho\to\infty$ plus gravitational field $\varphi_{\mu\nu}$

Under a coordinate transformation $\delta_\chi\sigma^\mu = \chi^\mu$

$$\varphi^{(R)}_{\mu\nu} = \varphi_{\mu\nu} + \nabla_\mu\chi_\nu + \nabla_\nu\chi_\mu \quad \text{where } \varphi_{\mu\nu} = \frac{\partial x^\alpha}{\partial\sigma^\mu}\frac{\partial x^\beta}{\partial\sigma^\mu}h_{\alpha\beta}$$

Rindler Gauge: $\quad\varphi^{(R)}_{\rho\mu} = 0$ implies four equations to solve χ_μ

- Asymptotic conditions around $x^- = 0$

$$\varphi^{(R)}_{\mu\nu} = \begin{pmatrix} \mathcal{O}((x^-)^2) & 0 & \mathcal{O}(x^-) & \mathcal{O}(x^-) \\ 0 & 0 & 0 & 0 \\ \mathcal{O}(x^-) & 0 & \mathcal{O}((x^-)^0) & \mathcal{O}((x^-)^0) \\ \mathcal{O}(x^-) & 0 & \mathcal{O}((x^-)^0) & \mathcal{O}((x^-)^0) \end{pmatrix}$$

- Any weak field takes the previous form around the Rindler horizon $x^- = 0$
- We found χ_μ such as incoming gravitational field takes nonzero values in a region $x^+\in[x_i^+, x_f^+]$ with $x_i^+ > 0$

3) Regge-Teitelboim canonical theory results

- The ADM decomposition of the metric is given $\qquad a = \{\rho, y, z\}$

$$ds^2 = -N^2d\tau^2 + h_{ab}\left(d\sigma^a + N^a d\tau\right)\left(d\sigma^b + N^b d\tau\right)$$

- Under an infinitesimal transformation $\delta_\xi\sigma^\mu = \xi^\mu(\tau,\rho,y,z)$, the generator of asymptotic symmetry $G[\xi] = H[\xi] + Q[\xi]$. But Einstein equations imply $H[\xi] = 0$.
- $Q[\xi]$ Integrability is nontrivial

$$\delta Q[\xi] = \int d^2S_\rho\left[G^{abpd}\left(\hat{\xi}^\tau\delta h_{ab|d} - \hat{\xi}^\tau{}_{|d}\delta h_{ab}\right) + 2\hat{\xi}_a\delta\Pi^{a\rho} - \hat{\xi}^\rho\Pi^{ab}\delta h_{ab}\right]$$

- A Rindler spacetime admits the integrability of the asymptotic transformation

$$\tau' = \tau + T(y,z) \quad \rho' = \rho \quad x'_A = X_A(y,z)$$

- Restricting to linear gravity in the pure gauge region $x^+ > x^+_f$ the main results are:

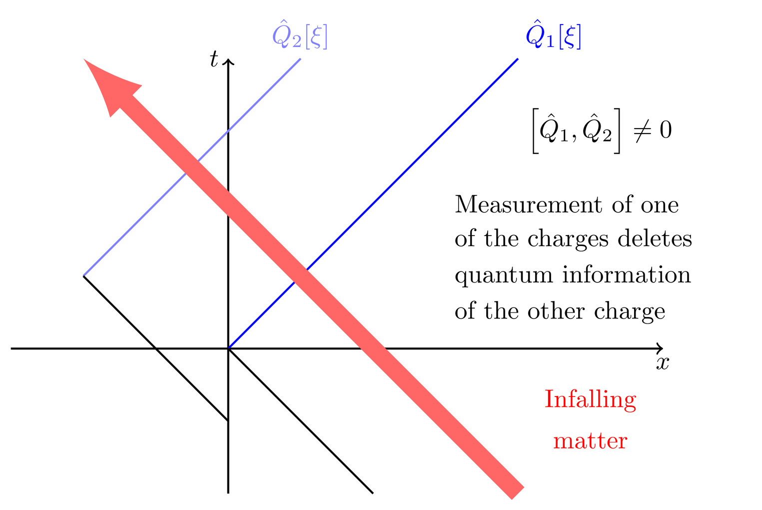
$$Q_{ST}[\xi^\tau] = -\tfrac{1}{2\kappa^2}\int dydz\ \xi^\tau(y,z)\left[\int_0^\infty x^+T_{++}(x^+,0,y,z)dx^+\right]$$

$$Q_{SR}[\xi^A] = \tfrac{1}{4\pi}\int dydz\ \xi^A \int dy'dz'\partial_A\left[\ln\left(\tfrac{(y-y')^2+(z-z')^2}{\kappa^2}\right)\int_0^\infty dx^+\partial_- T_{++}(x^+,0,y',z')\right]$$

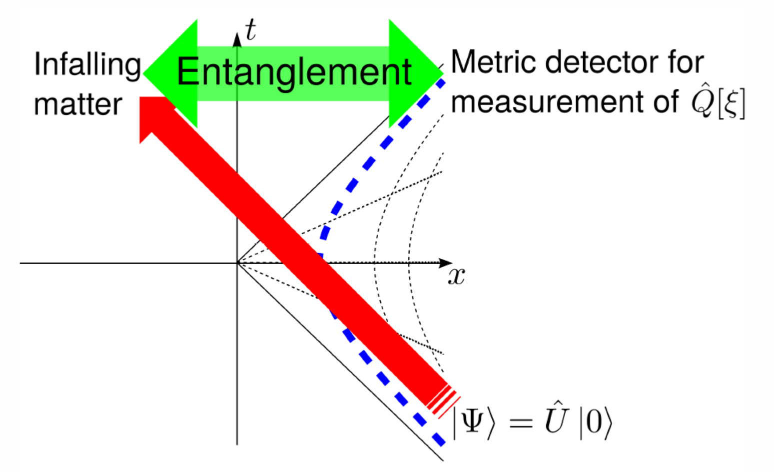
where $T_{\alpha\beta}$ is the matter-energy momentum tensor. Notice that if $T_{\alpha\beta} = 0$ then $Q = 0$

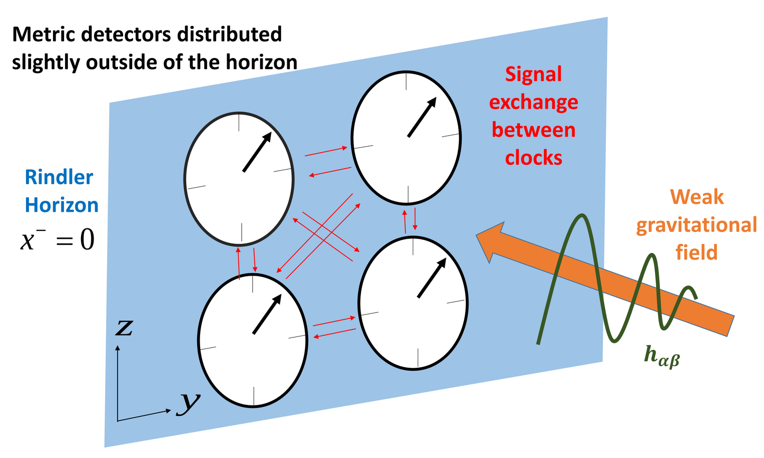
- If we contruct a Memory Operator $\hat{Q}$, then the resolution of no cloning paradox:

Between two quantum memories



When matter is absorbed





Metric detectors are required to measure the information stored in Q

The detectors inevitably interact with the infalling matter and share entanglement.

This causes decoherence of the infalling matter and no duplicate quantum information

4) Results with the Wald Zoupas Formulation

- ξ^μ a set of vector fields that preserve the asymptotic symmetries.

$$Q[\xi] = \oint_{\partial\Sigma}(\mathrm{d}^{d-2}x)_{\mu\nu}\left(Q^{\mu\nu}[\xi] - 2B^{[\mu}\xi^{\nu]}\right)$$

$$\delta\oint_{\partial\Sigma}(\mathrm{d}^{d-2}x)_{\mu\nu}\xi^{[\mu}B^{\nu]} = \oint_{\partial\Sigma}(\mathrm{d}^{d-2}x)_{\mu\nu}\xi^{[\mu}\Theta^{\nu]}(g,\delta g)$$

- From the Einstein-Hilbert action with $\kappa = \sqrt{16\pi G}$

$$Q^{\mu\nu} = -\frac{\sqrt{-\det g}}{\kappa^2}\left(\nabla^\mu\xi^\nu - \nabla^\nu\xi^\mu\right) \qquad \Theta^\nu = \frac{\sqrt{-\det g}}{\kappa^2}\left(g^{\nu\alpha}\nabla^\beta\delta g_{\alpha\beta} - g^{\alpha\beta}\nabla^\nu\delta g_{\alpha\beta}\right)$$

- When we consider Rindler horizons plus a gravitational field

$$Q^{u\rho} = \frac{\sqrt{f_0^2\det\Omega}}{\kappa^2 f_0^2}\left[-\xi^u\left(\frac{\partial}{\partial u} + \frac{1}{\kappa}\right)f_0 + \xi^A\left(\left(\frac{\partial}{\partial u} + \frac{1}{\kappa}\right)f_A - \frac{\partial\Omega_{BA}}{\partial u}f_C\Omega^{CB}\right)\right]$$

$$\Theta^\rho = \frac{\sqrt{\det\Omega}}{\kappa^2}\left[-\frac{1}{2}\Omega^{AB}\frac{\partial\Omega_{AB}}{\partial u}\left(\frac{\delta f_0}{f_0}\right) + \frac{\partial}{\partial u}\left(\frac{\delta f_0}{f_0}\right) + \Omega^{AB}\frac{\partial\delta\Omega_{AB}}{\partial u} + \frac{1}{2}\frac{\partial\Omega^{AB}}{\partial u}\delta\Omega_{AB}\right]$$

where $u = \tau - \rho$ and we defined:

$$f_0(u,y,z) = 1 + \mathcal{O}(\Delta) \qquad f_A(u,y,z) = \frac{1}{\Delta}\varphi^{(R)}_{\tau A} \quad \Omega_{AB} = \delta_{AB} + \varphi^{(R)}_{AB}$$

- SR charges are integrable to all orders of the gravitational field $\varphi_{\mu\nu}$

$$Q_{SR}[\xi] = \lim_{x^-\to 0}\int \mathrm{d}y\ \mathrm{d}z\ \frac{\sqrt{\det\Omega}}{\kappa^2}\xi^A\left(\left(\frac{\partial}{\partial u} + \frac{1}{\kappa}\right)f_A - \frac{\partial\Omega_{AB}}{\partial u}\Omega^{BC}f_C\right)$$

- Same results as with Regge-Teitelboim when we consider only linear gravity

5) Conclusions

- At leading order no information of gravitational waves is stored by the charges
- The holographic charges are merely an emergent concept, whose physical meaning is conditioned to measurements by appropriate near-horizon metric measurement devices