Gravitational Memory Charges of Supertranslation and Superrotations on Rindler Horizons

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Masahiro Hotta, Koji Yamaguchi

Jose Daniel Trevison Solano

Graduate School of Science
Tohoku University
Department of Physics
Particle Theory and Cosmology Group
Sendai-Japan

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Outline

1. Introduction
2. Charges on Rindler Horizons
3. Quantum Memory operators
4. Conclusions and future work

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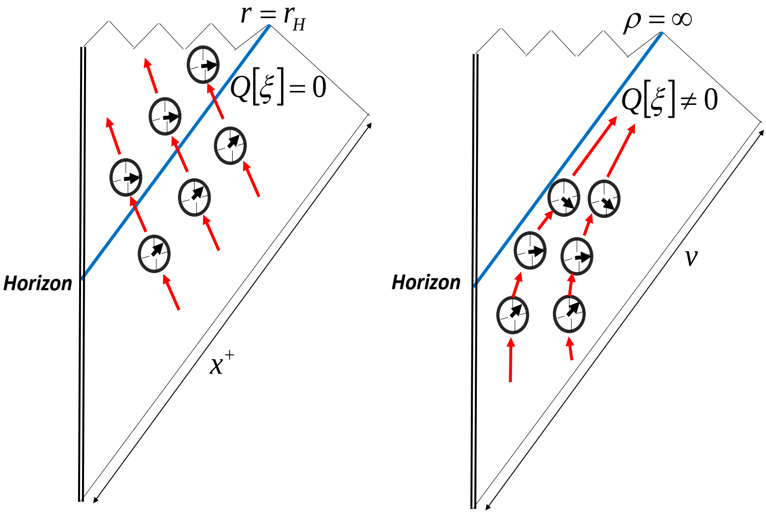
Supertranslations and Superrotations

- Hawking, Perry, and Strominger (HPS) proposed an scenario that may resolve the information loss problem (Phys. Rev. Lett. 116, 231301, 2016)
- They suggested that quantum information about collapsing matter is stored in an infinite number of conserved Noether currents having asymptotic symmetries (supertranslation on a horizon)
- The physical degrees of freedom emerge from would-be gauge degrees of freedom of the general covariance.

Hawking et al vs Us

It is not known yet whether a gravitational wave excites charged states of supertranslation and superrotation



Conclusions in Advance

- We construct a general theory of gravitational holographic charges for a (1+3)-dimensional linearized gravity field in the Minkowski background with Rindler horizons.

- In linear gravity approximation no information of gravitational waves is by the charges

- The holographic charges are merely an emergent concept, whose physical meaning is conditioned to measurements by appropriate near-horizon metric measurement devices

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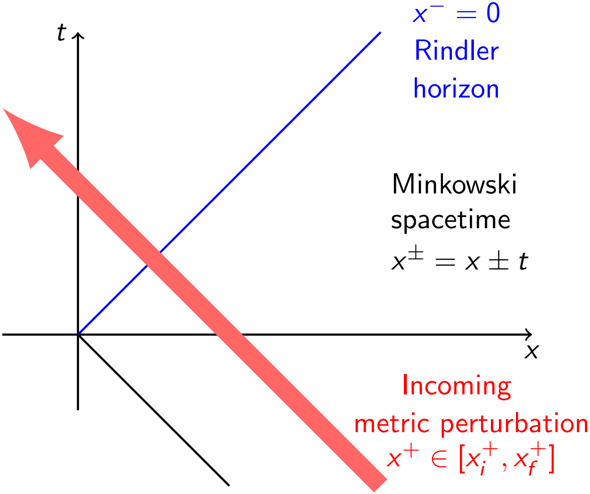
Our model

$$ds^2 = dx^+ dx^- + dy^2 + dz^2 + h_{\alpha\beta} dx^\alpha dx^\beta$$



Rindler Gauge

- Rindler coordinates: $\sigma^\mu = \{\tau, \rho, y, z\}$

$$x^\pm = 2\kappa \exp\left(-\frac{\rho \mp \tau}{2\kappa}\right) \qquad \text{with } \Delta = \exp\left(-\frac{\rho}{\kappa}\right) = \frac{x^+ x^-}{4\kappa^2}$$

$$ds^2 = \Delta\left(-d\tau^2 + d\rho^2\right) + dy^2 + dz^2 + \varphi_{\mu\nu} d\sigma^\mu d\sigma^\nu$$

- Under an infinitesimal coordinate transformation $\delta_\Theta \sigma^\mu = \Theta^\mu$

$$\varphi^{(R)}_{\mu\nu} = \varphi_{\mu\nu} + \nabla_\mu \Theta_\nu + \nabla_\nu \Theta_\mu \qquad \text{where } \varphi_{\mu\nu} = \frac{\partial x^\alpha}{\partial \sigma^\mu} \frac{\partial x^\beta}{\partial \sigma^\mu} h_{\alpha\beta}$$

- Gauge Fixing: $\varphi^{(R)}_{\rho\mu} = 0$ implies four equations to solve Θ_μ

Rindler Gauge

- Asymptotic conditions around $x^- = 0$

$$\varphi^{(R)}_{\mu\nu} = \begin{pmatrix} \mathcal{O}((x^-)^2) & 0 & \mathcal{O}(x^-) & \mathcal{O}(x^-) \\ 0 & 0 & 0 & 0 \\ \mathcal{O}(x^-) & 0 & \mathcal{O}((x^-)^0) & \mathcal{O}((x^-)^0) \\ \mathcal{O}(x^-) & 0 & \mathcal{O}((x^-)^0) & \mathcal{O}((x^-)^0) \end{pmatrix}$$

- Any weak field takes the previous form around the Rindler horizon $x^- = 0$

- We found Θ_μ such as incoming gravitational field takes nonzero values in a region $x^+ \in [x_i^+, x_f^+]$ with $x_i^+ > 0$

Regge-Teitelboim canonical theory

- The ADM decomposition of the metric is given

$$ds^2 = -N^2 d\tau^2 + h_{ab}\Big(d\sigma^a + N^a d\tau\Big)\Big(d\sigma^b + N^b d\tau\Big)$$

  where $a = \{\rho, y, z\}$

- The conjugate momentum Π^{ab} of h_{ab} can be defined
- The Hamiltonian density $\mathcal{H}$ and momentum density $\mathcal{H}_a$ can also be defined
- Einstein equations imposes the constraints:

$$\mathcal{H} = 0 \qquad\qquad \mathcal{H}_a = 0$$

Generator of coordinate transformations

- Infinitesimal transformation $\delta_\xi \sigma^\mu = \xi^\mu(\tau, \rho, y, z)$
- The generator $G[\xi]$ is the sum of two terms: $G[\xi] = H[\xi] + Q[\xi]$
- If the equations of motion are satisfied $H[\xi] = 0$
- The term $Q[\xi]$ is obtained by integration of

$$\delta Q[\xi] = \int d^3\sigma \left[\mathcal{A}\left(h, \Pi, \xi\right) \delta h + \mathcal{B}\left(h, \Pi, \xi\right) \delta \Pi\right]$$

- Integrability is nontrivial
- A Rindler spacetime admits the integrability of the asymptotic transformation $A = \{y, z\}$

$$\tau' = \tau + T(y, z) \qquad \rho' = \rho \qquad x'_A = X_A(y, z)$$

Main Result

- Supertranslations: $\xi^\tau = \xi^\tau(y,z) \qquad \xi^\rho = 0 \qquad \xi^A = 0$

$$Q_{st}[\xi^\tau] = -\frac{1}{2\kappa^2} \lim_{x^- \to 0} \int dydz\ \xi^\tau \left[x^+ \partial_+ + x^- \partial_- - 1\right] h^{(R)}_{AA}(x^+, x^-, y, z)$$

- Superrotations: $\xi^\tau = 0 \qquad \xi^\rho = 0 \qquad \xi^A = \xi^A(y,z)$

$$Q_{sr}[\xi^A] = -\frac{2}{\kappa x^+} \lim_{x^- \to 0} \int dydz\ \xi^A \left[x^+ \partial_+ + x^- \partial_- + 1\right] h^{(R)}_{-A}(x^+, x^-, y, z)$$

where $h^{(R)}_{\alpha\beta} = h_{\alpha\beta} + \partial_\alpha \theta_\beta + \partial_\beta \theta_\alpha$

Main Result

- By using Einstein equations in the pure gauge region $x^+ > x_f^+$
- Supertranslations: $\xi^\tau = \xi^\tau(y,z) \qquad \xi^\rho = 0 \qquad \xi^A = 0$

$$Q_{st}[\xi^\tau] = -\frac{1}{2\kappa^2}\int dydz\ \xi^\tau\left[\int_0^\infty x^+ T_{++}(x^+,0,y,z)dx^+\right]$$

- Superrotations: $\xi^\tau = 0 \qquad \xi^\rho = 0 \qquad \xi^A = \xi^A(y,z)$

$$Q_{sr}[\xi^A] = +\frac{1}{4\pi}\int dydz\ \xi^A \int dy'dz' \qquad \partial_A\left[\ln\left(\frac{(y-y')^2+(z-z')^2}{\kappa^2}\right)\right.$$
$$\left.\int_0^\infty dx^+\partial_- T_{++}(x^+,0,y',z')\right]$$

where $T_{\alpha\beta}$ is the matter-energy momentum tensor

Result analysis

- Weak gravitational waves: $T_{++} = 0 \Longrightarrow Q[\xi] = 0$
- The classical energy condition ensures positivity of the generator of the Lorentz boost (Rindler energy)

$$E_R = \int_0^\infty x^+ T_{++}(x^+, 0, y, z) \geq 0$$

  The local supertranslation charge retains only information about the total amount of E_R that passes through the point (y,z)

- $Q_{sr}[\xi^A]$ stores information about matter nonlocally

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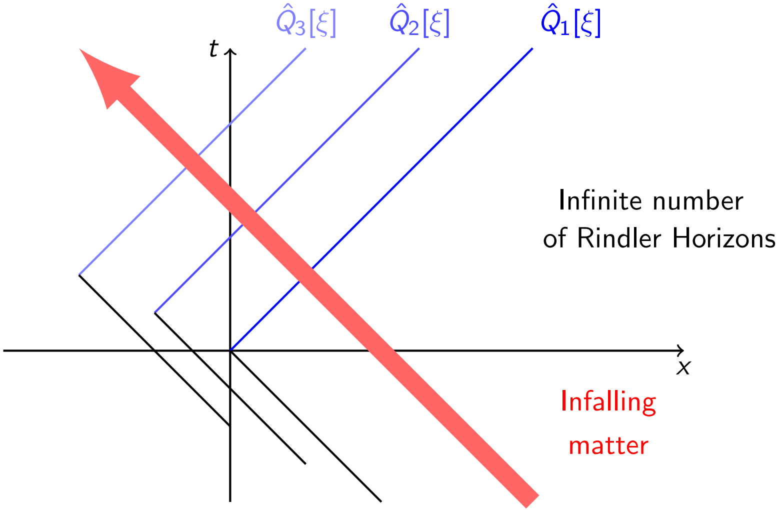
Information shared by multiple horizons?



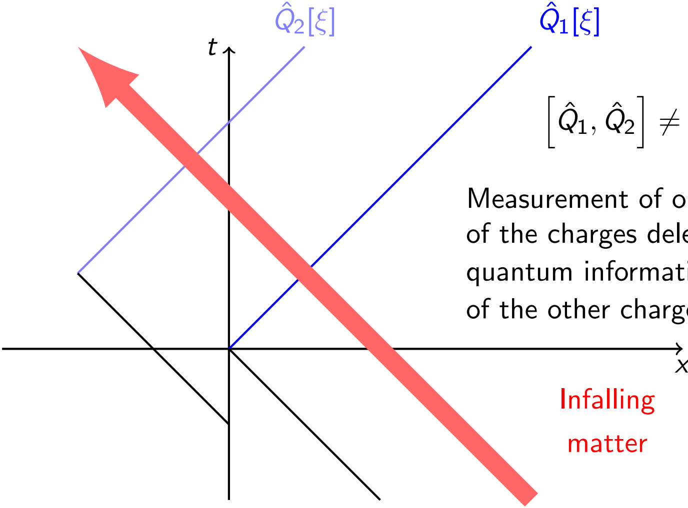
Noncommutativity of the charges

- Let us define the quantum gravitational memory operators on a future horizon at $x^- = x_h^-$ with a Rindler wedge located at $(x^+, x^-) = (x_h^+, x_h^-)$ as

$$\begin{aligned}\hat{Q}(x_h^+, x_h^-) &= -\frac{1}{2\kappa}\int dydz\ \xi^\tau \left[\int_0^\infty dx^+ \left(x^+ + x_h^+\right)\ \hat{T}_{++}(x^+ + x_h^+, x_h^-, y, z)\right] \\ &\quad +\frac{1}{4\pi}\int dydz\ \xi^A \int dy' dz' \partial_A \left[\ln\left(\frac{(y-y')^2 + (z-z')^2}{\kappa^2}\right)\right. \\ &\qquad \left.\int_0^\infty dx^+ \partial_- \hat{T}_{++}(x^+ + x_h^+, x_h^-, y', z')\right]\end{aligned}$$

- $\left[\hat{T}_{++}(X), \hat{T}_{++}(Y)\right] \neq 0$ then $\left[\hat{Q}(x_h^+, x_h^-), \hat{Q}(x_h'^+, x_h'^-)\right] \neq 0$

Noncommutativity of the charges

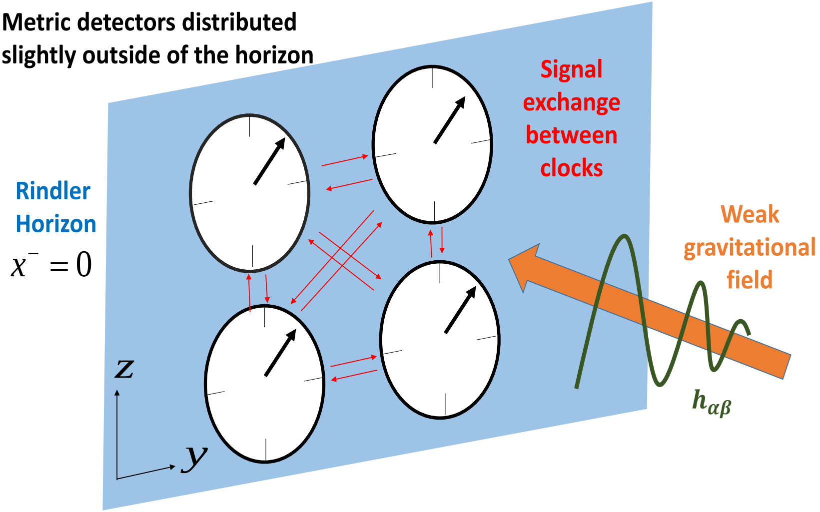


$$\left[\hat{Q}_1, \hat{Q}_2\right] \neq 0$$

Measurement of one of the charges deletes quantum information of the other charge

Charge is conditioned by measurements

Without measurement devices the charge is merely a gauge freedom of the general covariance in Minkowski background



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Status and Future Work

- Second order superrotations has already been calculated
- Simple models of metric detectors in (1+1) dimensions have been proposed
- To do list
  1. Metric detectors in 1+3 dimensions
  2. Calculation of second order supertranslations

Appendix: Covariant current formulation

- ξ a set of vector fields that preserve the symmetry to study, in this case asymptotic symmetries.

$$Q[\xi] = \oint_{\partial\Sigma} (\mathrm{d}^{d-2}x)_{\mu\nu} \left(Q^{\mu\nu}[\xi] - 2B^{[\mu}\xi^{\nu]} \right)$$

$$\delta \oint_{\partial\Sigma} (\mathrm{d}^{d-2}x)_{\mu\nu} \xi^{[\mu} B^{\nu]} = \oint_{\partial\Sigma} (\mathrm{d}^{d-2}x)_{\mu\nu} \xi^{[\mu} \Theta^{\nu]}(g, \delta g)$$

- From the Einstein-Hilbert action with $\kappa = \sqrt{16\pi G}$

$$\begin{aligned} Q^{\mu\nu} &= -\frac{\sqrt{-\det g}}{\kappa^2} \left(\nabla^\mu \xi^\nu - \nabla^\nu \xi^\mu \right) \\ \Theta^\nu &= \frac{\sqrt{-\det g}}{\kappa^2} \left(g^{\nu\alpha} \nabla^\beta \delta g_{\alpha\beta} - g^{\alpha\beta} \nabla^\nu \delta g_{\alpha\beta} \right) \end{aligned}$$